Summary presentation on LSIDR(s), local pseudo-inversion, preconditioning for local pseudo-inversion

–

Martin Neuenhofen, 26.02.2015

Part 1:

LSIDR(s)

LSIDR(s): idea

- One Kaczmarz-cycle orthogonalizes a residual on each column vector of the system matrix in a fixed order.
- As the column vectors themself are not orthogonal on each other, the residual usually will also not be othogonal on all of them after the Kaczmarz-cycle.
- The Kaczmarz-cycle can be written as a matrix-vector-product. The referring unsymmetric singular matrix is called Kaczmarz-matrix or filter-matrix.
- Notice: A forward and backward Kaczmarz-cycle gives a symmetric positive semi-definite matrix, which has already been used, e.g. in CARP-CG, KACZ-CG and KACZ-MINRES.

LSIDR(s): idea

- The idea is to search a residual in the Krylov space of this filter-matrix
- For m-by-n-matrices, this Krylov space has n dimensions for any fixed right-hand-side b.
- Use the IDR(s) principle to select the residual in spaces G_i of shrinking dimension and choose omega to minimize length of the residual
- Observation: As one iteration is as computational intensive as one CGNR step, LSIDR(s) seems to converge faster for several applications, where it needs less iterations

LSIDR(s): difficulties

- The number of dimensions of the Krylov space of the filter matrix is so to say unstable.
  - Due to round of errors there occour residual directions which are not in span(A,b).
  - To overcome this, one is recommended to incorporate so called resynchronization computations.
- IDR(s) principle needs a little adaption for this.
  - You have to choose Sonneveld vectors in the span of the system.
  - To intersect your residual in G with S, you have have to choose special auxiliary vectors to conserve a length property of the residual.

Part 2:
local pseudo-inversion

local pseudo-inversion: idea

- A pseudo-inverse row can be computed as residual of a least-squares solution.
- Any residual minimizing method works for local pseudo-inversion as error minimizing method.
- For column rank deficient systems this strategy also works under specific conditions.
- local pseudo-inversion may have advantages over global solution
  - efficient symmetry exploiting preconditioning
  - solution in one unknown for any right-hand-side in complexity of one scalar product

Part 3:
preconditioning for local pseudo inversion

preconditioning: idea

- If
  - your operator and your grid is symmetric (e.g. Poisson on circle region, finite differences in polar coordinates)
  - and the local point, in which you want to solve, is located on symmetry lines of the grid
  - and only if you use a Krylov method based on the filter matrix, e.g. LSIDR(s)
- then
  - you can exploit the symmetry of your problem by reducing
    - cost per iteration
    - number of dimensions of the Krylov space
    - error reduction per iteration

preconditioning: idea

- The principle is to filter the residual on a subset of equations, where each equation refers to one node of the grid, then to index permutation to generate a tupel, which is the filtered vector on another subset of nodes.
- Recombine both vectors in optimal way by averaging.
- The dimension of Krylov space halves for each symmetry line that you exploit
- The cost approximately also halves, logarithmic cost in n neglected.